CS 335: Bottom-up Parsing

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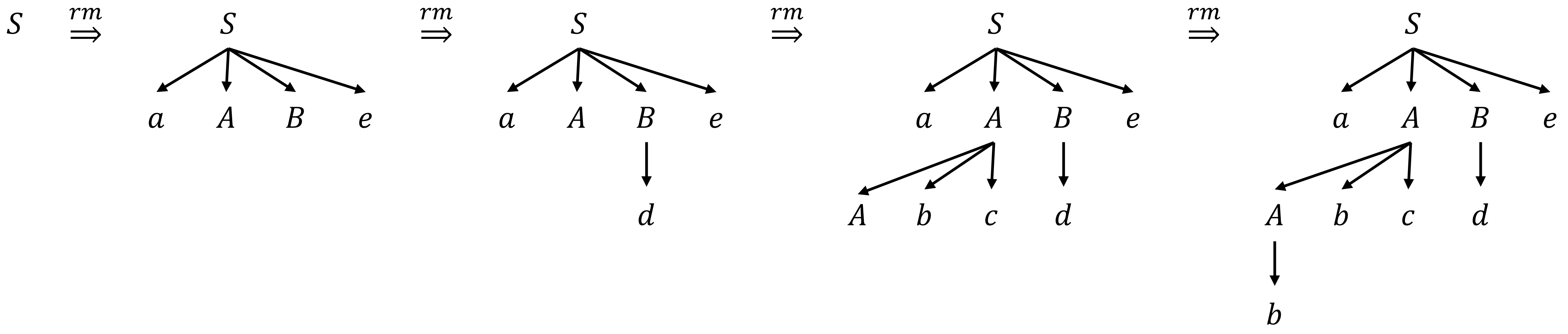
Content influenced by many excellent references, see References slide for acknowledgements.

Rightmost Derivation of $abbcde$

$S \rightarrow aABe$
$A \rightarrow Abc \mid b$
$B \rightarrow d$

Input string: $abbcde$
$S \rightarrow aABe$
$\rightarrow aAde$
$\rightarrow aAbcde$
$\rightarrow abbcde$



Bottom-up Parsing

Constructs the parse tree starting from the leaves and working up toward the root

$S \rightarrow aABe$
$A \rightarrow Abc \mid b$
$B \rightarrow d$

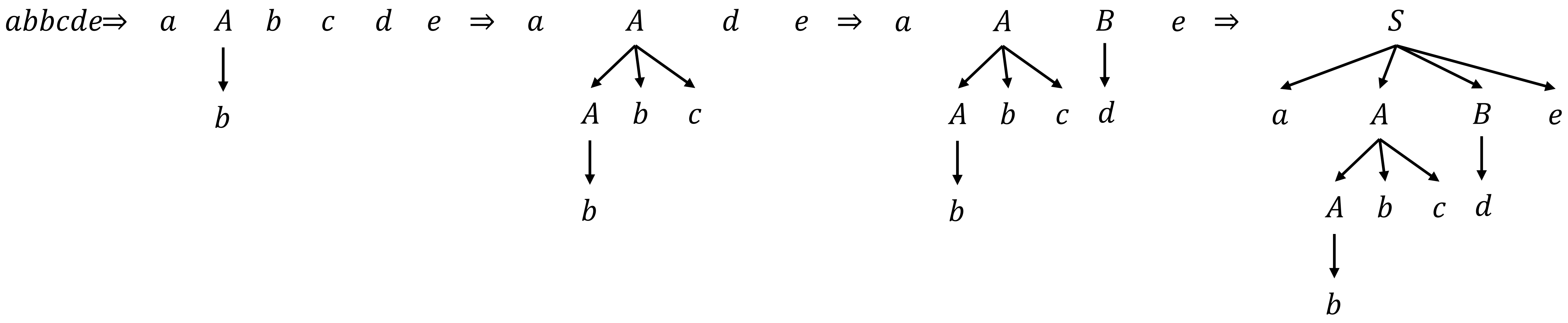
Input string: $abbcde$	
$S \rightarrow aABe$	$abbcde$
$\rightarrow aAde$	$\rightarrow aAbcde$
$\rightarrow aAbcde$	$\rightarrow aAde$
$\rightarrow abbcde$	$\rightarrow aABe$
	$\rightarrow S$

reverse of rightmost derivation

Bottom-up Parsing

$S \rightarrow aABe$
$A \rightarrow Abc \mid b$
$B \rightarrow d$

Input string: $abbcde$
$abbcde$
$\rightarrow aAbcde$
$\rightarrow aAde$
$\rightarrow aABe$
$\rightarrow S$



Reduction

- Bottom-up parsing **reduces** a string w to the start symbol S
  - At each reduction step, a chosen substring that is the rhs (or body) of a production is replaced by the lhs (or head) nonterminal

Derivation

$$\boxed{S \underset{rm}{\Rightarrow} \gamma_0 \underset{rm}{\Rightarrow} \gamma_1 \underset{rm}{\Rightarrow} \gamma_2 \underset{rm}{\Rightarrow} \ldots \underset{rm}{\Rightarrow} \gamma_{n-1} \underset{rm}{\Rightarrow} \gamma_n = w}$$

Bottom-up Parser

Handle

- Handle is a substring that matches the body of a production
  - Reducing the handle is one step in the reverse of the rightmost derivation

$E \rightarrow E + T \mid T$
$T \rightarrow T * F \mid F$
$F \rightarrow (E) \mid \textbf{id}$

Right Sentential Form	Handle	Reducing Production
$\textbf{id}_1 * \textbf{id}_2$	$\textbf{id}_1$	$F \rightarrow \textbf{id}$
$F * \textbf{id}_2$	F	$T \rightarrow F$
$T * \textbf{id}_2$	$\textbf{id}_2$	$F \rightarrow \textbf{id}$
$T * F$	$T * F$	$T \rightarrow T * F$
T	T	$E \rightarrow T$

Handle

Although T is the body of the production $E \rightarrow T$, T is not a handle in the sentential form $T * \textbf{id}_2$

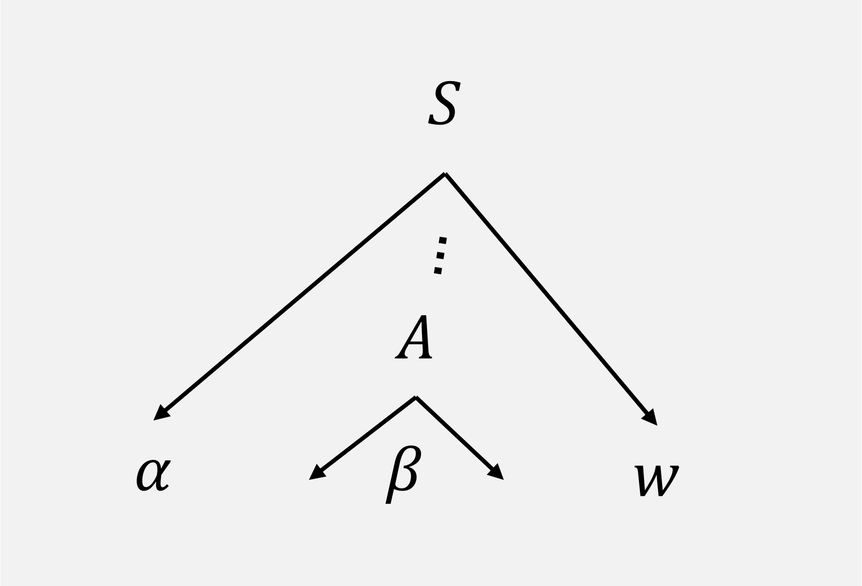
$E \rightarrow E + T \mid T$
$T \rightarrow T * F \mid F$
$F \rightarrow (E) \mid \textbf{id}$

Right Sentential Form	Handle	Reducing Production
$\textbf{id}_1 * \textbf{id}_2$	$\textbf{id}_1$	$F \rightarrow \textbf{id}$
$F * \textbf{id}_2$	F	$T \rightarrow F$
$T * \textbf{id}_2$	$\textbf{id}_2$	$F \rightarrow \textbf{id}$
$T * F$	$T * F$	$T \rightarrow T * F$
T	T	$E \rightarrow T$

Handle

- If $S \underset{rm}{\Rightarrow}^* \alpha A w \underset{rm}{\Rightarrow} \alpha\beta w$, then $A \rightarrow \beta$ is a handle of $\alpha\beta w$

- String w right of a handle must contain only terminals



A handle $A \rightarrow \beta$ in the parse tree for $\alpha\beta w$

Handle

If grammar G is unambiguous, then every right sentential form has only one handle

If G is ambiguous, then there can be more than one rightmost derivation of $\alpha\beta w$

Shift-Reduce Parsing

Shift-Reduce Parsing

- Type of bottom-up parsing with two primary actions, shift and reduce
  - Other obvious actions are accept and error

- The input string (i.e., being parsed) consists of two parts
  - Left part is a string of terminals and nonterminals, and is stored in stack
  - Right part is a string of terminals read from an input buffer
  - Bottom of the stack and end of input are represented by \$

Shift-Reduce Actions

- **Shift**: shift the next input symbol from the right string onto the top of the stack
- **Reduce**: identify a string on top of the stack that is the body of a production, and replace the body with the head

Shift-Reduce Parsing

- **Initial**

Stack	Input
\$	$w\$$

Shift $\Downarrow *$ Reduce

- **Final goal**

Stack	Input
$\$S$	\$

Shift-Reduce Parsing

$E \rightarrow E + T \mid T$
$T \rightarrow T * F \mid F$
$F \rightarrow (E) \mid \mathbf{id}$

Stack	Input	Action
$\$$	$\mathbf{id}_1 * \mathbf{id}_2\$$	Shift
$\$\mathbf{id}_1$	$* \mathbf{id}_2\$$	Reduce by $F \rightarrow$ id
$\$F$	$* \mathbf{id}_2\$$	Reduce by $T \rightarrow F$
$\$T$	$* \mathbf{id}_2\$$	Shift
$\$T *$	$\mathbf{id}_2\$$	Shift
$\$T * \mathbf{id}_2$	$\$$	Reduce by $F \rightarrow$ id
$\$T * F$	$\$$	Reduce by $T \rightarrow T * F$
$\$T$	$\$$	Reduce by $E \rightarrow T$
$\$E$	$\$$	Accept

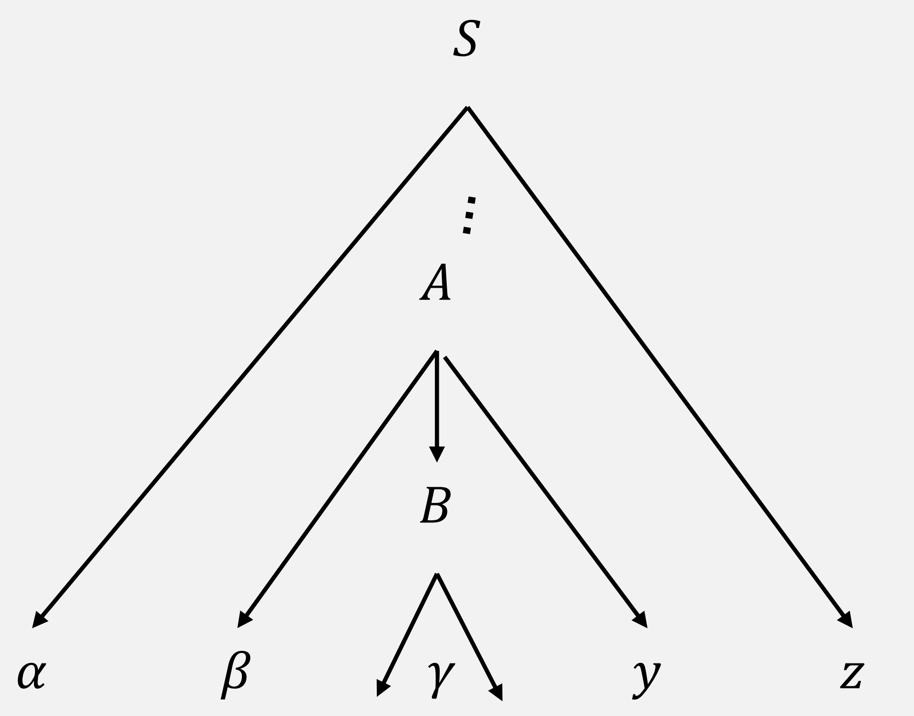
Handle on Top of the Stack

- Is the following scenario possible?

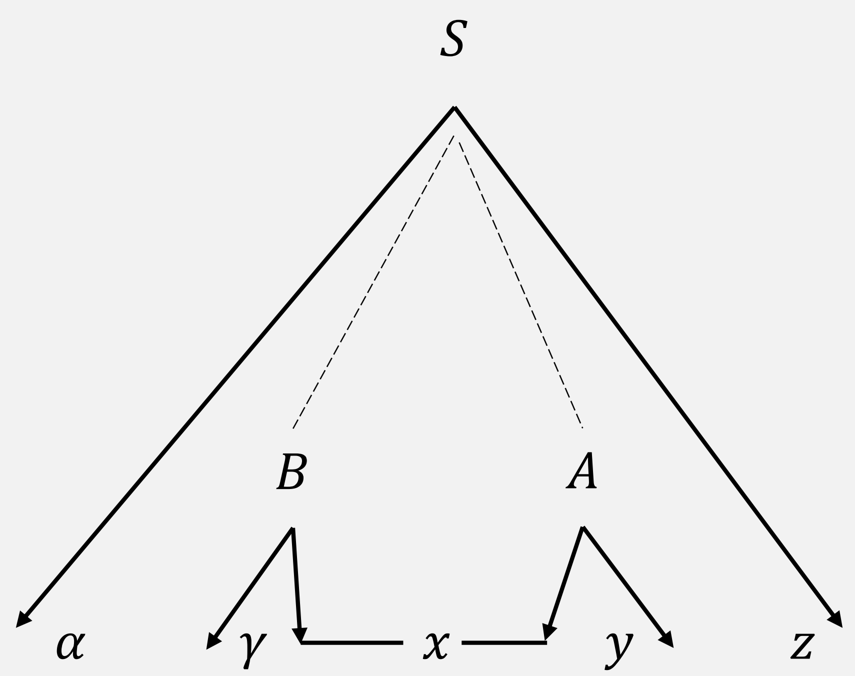
Stack	Input	Action
…		
$\$ \alpha\beta\gamma$	$w\$$	Reduce by $A \rightarrow \gamma$
$\$ \alpha\beta A$	$w\$$	Reduce by $B \rightarrow \beta$
$\$\alpha BA$	$w\$$	
…		

Possible Choices in Rightmost Derivation

1. $S \underset{rm}{\Rightarrow} \alpha Az \underset{rm}{\Rightarrow} \alpha\beta Byz \underset{rm}{\Rightarrow} \alpha\beta\gamma yz$



2. $S \underset{rm}{\Rightarrow} \alpha BxAz \underset{rm}{\Rightarrow} \alpha Bxyz \underset{rm}{\Rightarrow} \alpha\gamma xyz$



Handle on Top of the Stack

- Is the following scenario possible?

Stack	Input	Action
…		

Handle always eventually appears on top of the stack, never inside

Shift-Reduce Actions

- Shift: shift the next input symbol from the right string onto the top of the stack
- Reduce: identify a string on top of the stack that is the body of a production, and replace the body with the head

How do you decide when to shift and when to reduce?

Steps in Shift-Reduce Parsers

General shift-reduce technique

If there is **no handle** on the stack, **then shift**

If there is **a handle** on the stack, **then reduce**

- Bottom up parsing is essentially the process of detecting handles and reducing them
- Different bottom-up parsers differ in the way they detect handles

Challenges in Bottom-up Parsing

Which action do you pick when there is a choice?

- Both shift and reduce are valid, implies a shift-reduce conflict

Which rule to use if reduction is possible by more than one rule?

- Reduce-reduce conflict

Shift-Reduce Conflict

$E \rightarrow E + E \mid E * E \mid \mathbf{id}$

$\mathbf{id} + \mathbf{id} * \mathbf{id}$

Stack	Input	Action
$\$$	$\mathbf{id} + \mathbf{id} * \mathbf{id}\$$	Shift
...		
$\$E + E$	$* \mathbf{id}\$$	Reduce by $E \rightarrow E + E$
$\$E$	$* \mathbf{id}\$$	Shift
$\$E *$	$\mathbf{id}\$$	Shift
$\$E * \mathbf{id}$	$\$$	Reduce by $E \rightarrow \mathbf{id}$
$\$E * E$	$\$$	Reduce by $E \rightarrow E * E$
$\$E$	$\$$	

$\mathbf{id} + \mathbf{id} * \mathbf{id}$

Stack	Input	Action
$\$$	$\mathbf{id} + \mathbf{id} * \mathbf{id}\$$	Shift
...		
$\$E + E$	$* \mathbf{id}\$$	Shift
$\$E + E *$	$\mathbf{id}\$$	Shift
$\$E + E * \mathbf{id}$	$\$$	Reduce by $E \rightarrow \mathbf{id}$
$\$E + E * E$	$\$$	Reduce by $E \rightarrow E * E$
$\$E + E$	$\$$	Reduce by $E \rightarrow E + E$
$\$E$	$\$$	

Shift-Reduce Conflict

$$
\begin{aligned}
Stmt \rightarrow & \ \textbf{if}\ Expr\ \textbf{then}\ Stmt \\
& \mid \textbf{if}\ Expr\ \textbf{then}\ Stmt\ \textbf{else}\ Stmt \\
& \mid other
\end{aligned}
$$

Stack	Input	Action
… **if** $Expr$ **then** $Stmt$	**else** … $	

Shift-Reduce Conflict

$$\begin{aligned} Stmt \rightarrow\ & \textbf{if}\ Expr\ \textbf{then}\ Stmt \\ |\ & \textbf{if}\ Expr\ \textbf{then}\ Stmt\ \textbf{else}\ Stmt \\ |\ & other \end{aligned}$$

Stack	Input	Action
... **if** $Expr$ **then** $Stmt$	**else** ... $\$$	

What is a correct thing to do for this grammar – shift or reduce?

Reduce-Reduce Conflict

$M \rightarrow R + R \mid R + c \mid R$
$R \rightarrow c$

$c + c$		
Stack	**Input**	**Action**
$\$$	$c + c\$$	Shift
$\$c$	$+c\$$	Reduce by $R \rightarrow c$
$\$R$	$+c\$$	Shift
$\$R +$	$c\$$	Shift
$\$R + c$	$\$$	Reduce by $R \rightarrow c$
$\$R + R$	$\$$	Reduce by $R \rightarrow R + R$
$\$M$	$\$$	

$c + c$		
Stack	**Input**	**Action**
$\$$	$c + c\$$	Shift
$\$c$	$+c\$$	Reduce by $R \rightarrow c$
$\$R$	$+c\$$	Shift
$\$R +$	$c\$$	Shift
$\$R + c$	$\$$	Reduce by $M \rightarrow R + c$
$\$M$	$\$$	

LR Parsing

LR(k) Parsing

- Popular bottom-up parsing scheme
  - L is for left-to-right scan of input
  - R is for reverse of rightmost derivation
  - k is the number of lookahead symbols
- LR parsers are table-driven, like the nonrecursive LL parser
- LR grammar is one for which we can construct an LR parsing table

Popularity of LR Parsing

Can recognize all language constructs with CFGs

Most general nonbacktracking shift-reduce parsing method

Works for a superset of grammars parsed with predictive or LL parsers

Why?

Popularity of LR Parsing

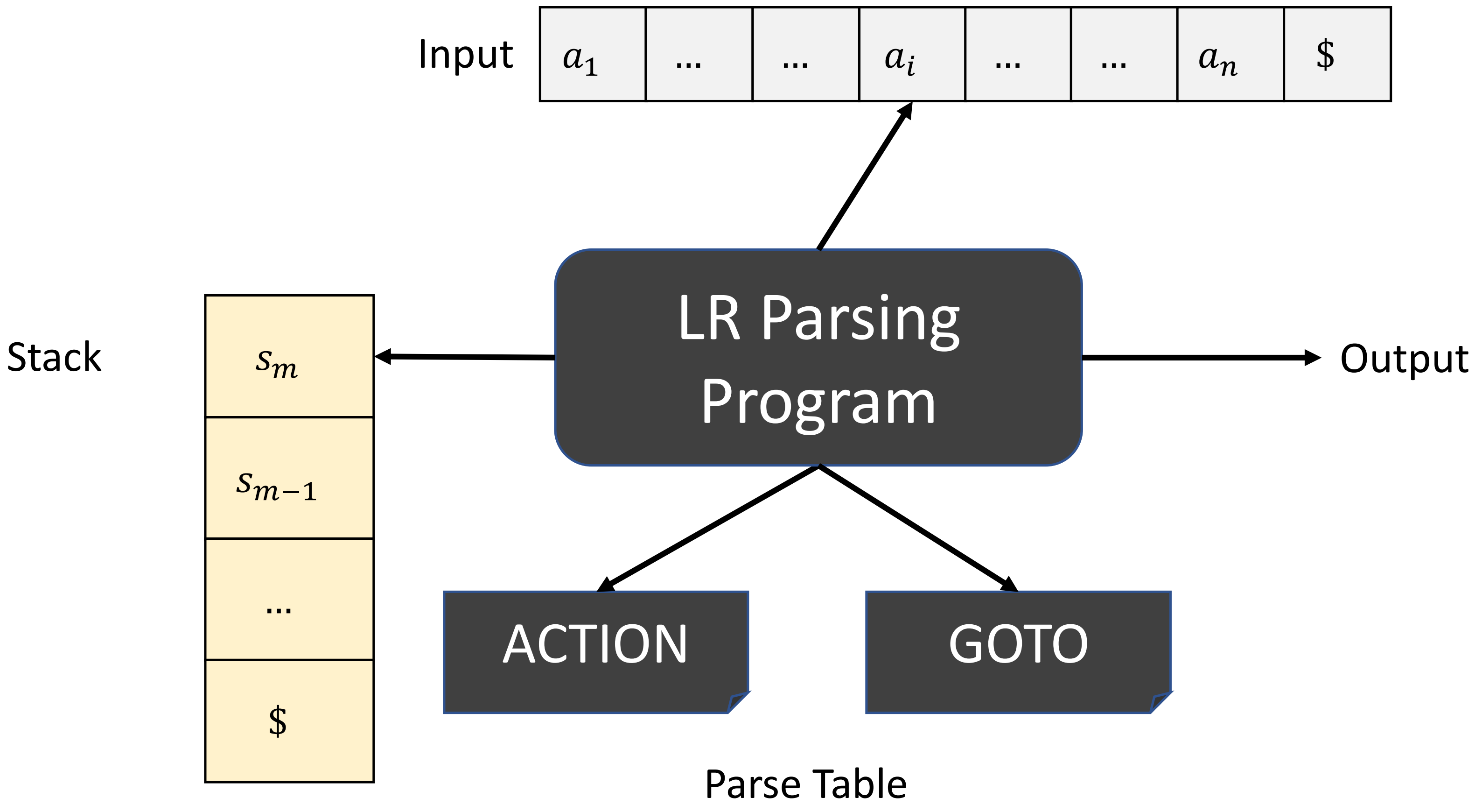
Can recognize all language constructs with CFGs

Most general nonbacktracking shift-reduce parsing method

Works for a superset of grammars parsed with predictive or LL parsers

- LL(k) parsing predicts which production to use having seen only the first k tokens of the right-hand side
- LR(k) parsing can decide after it has seen input tokens corresponding to the entire right-hand side of the production

Block Diagram of LR Parser



LR Parsing

- Remember the basic question: when to shift and when to reduce!
- Information is encoded in a DFA constructed using canonical LR(0) collection
  I. Augmented grammar G' with new start symbol S' and rule $S' \rightarrow S$
  II. Define helper functions Closure() and Goto()

LR(0) Item

- An LR(0) item (also called item) of a grammar G is a production of G with a dot at some position in the body
- An item indicates how much of a production we have seen
  - Symbols on the left of "•" are already on the stack
  - Symbols on the right of "•" are expected in the input

Production	Items
$A \rightarrow XYZ$	$A \rightarrow \bullet XYZ$
	$A \rightarrow X \bullet YZ$
	$A \rightarrow XY \bullet Z$
	$A \rightarrow XYZ \bullet$

$A \rightarrow \bullet XYZ$ indicates that we expect a string derivable from XYZ next on the input

Closure Operation

- Let I be a set of items for a grammar G
- Closure(I) is constructed by
  1. Add every item in I to Closure(I)
  2. If $A \rightarrow \alpha \bullet B\beta$ is in Closure(I) and $B \rightarrow \gamma$ is a rule, then add $B \rightarrow \gamma$ to Closure(I) if not already added
  3. Repeat until no more new items can be added to Closure(I)

Example of Closure

$E' \rightarrow E$
$E \rightarrow E + T \mid T$
$T \rightarrow T * F \mid F$
$F \rightarrow (E) \mid \mathbf{id}$

Suppose $I = \{E' \rightarrow \bullet E\}$, compute Closure($I$)

Example of Closure

$$E' \rightarrow E$$
$$E \rightarrow E + T \mid T$$
$$T \rightarrow T * F \mid F$$
$$F \rightarrow (E) \mid \mathbf{id}$$

Suppose $I = \{E' \rightarrow \bullet E\}$

Closure$(I) = \{$

$$E' \rightarrow \bullet E,$$
$$- - - - - - - - - - - -$$
$$E \rightarrow \bullet E + T,$$
$$E \rightarrow \bullet T,$$
$$T \rightarrow \bullet T * F,$$
$$T \rightarrow \bullet F,$$
$$F \rightarrow \bullet (E),$$
$$F \rightarrow \bullet \mathbf{id}$$

$\}$

Kernel and Nonkernel Items

- If one B-production is added to Closure(I) with the dot at the left end, then all B-productions will be added to the closure
- Kernel items
  - Initial item $S' \rightarrow \bullet S$, and all items whose dots are not at the left end
- Nonkernel items
  - All items with their dots at the left end, except for $S' \rightarrow \bullet S$

Goto Operation

- Suppose I is a set of items and X is a grammar symbol
- Goto(I, X) is the closure of set all items $[A \rightarrow \alpha X \bullet \beta]$ such that $[A \rightarrow \alpha \bullet X \beta]$ is in I
  - If I is a set of items for some valid prefix α, then Goto(I,X) is set of valid items for prefix αX

- Intuitively, Goto(I, X) defines the transitions in the LR(0) automaton
  - Goto(I, X) gives the transition from state I under input X

Example of Goto

$$E' \rightarrow E$$
$$E \rightarrow E + T \mid T$$
$$T \rightarrow T * F \mid F$$
$$F \rightarrow (E) \mid \textbf{id}$$

Suppose $I = \{$

$$E' \rightarrow E\bullet,$$

$$E \rightarrow E\bullet + T$$

$\}$

- Compute Goto(I, $+$)

Example of Goto

$$E' \rightarrow E$$
$$E \rightarrow E + T \mid T$$
$$T \rightarrow T * F \mid F$$
$$F \rightarrow (E) \mid \textbf{id}$$

Suppose $I = \{$

$E' \rightarrow E\bullet,$

$E \rightarrow E\bullet + T$

$\}$

Goto(I, +) = {

$E \rightarrow E + \bullet T,$

$T \rightarrow \bullet T * F,$

$T \rightarrow \bullet F,$

$F \rightarrow \bullet(E),$

$F \rightarrow \bullet\textbf{id}$

}

Canonical Collection of Sets of LR(0) Items

C = Closure($\{S' \rightarrow \bullet S\}$)

repeat

  for each set of items I in C

    for each grammar symbol X

      if Goto(I, X) is not empty and not in C

        add Goto(I, X) to C

until no new sets of items are added to C

Canonical Collection of Sets of LR(0) Items

$$
\begin{aligned}
&E' \rightarrow E \\
&E \rightarrow E + T \mid T \\
&T \rightarrow T * F \mid F \\
&F \rightarrow (E) \mid \mathbf{id}
\end{aligned}
$$

- Compute the canonical collection for the expression grammar

Canonical Collection of Sets of LR(0) Items

$I_0 = \text{Closure}(E' \rightarrow \bullet E) = \{$
$E' \rightarrow \bullet E,$
$E \rightarrow \bullet E + T,$
$E \rightarrow \bullet T,$
$T \rightarrow \bullet T * F,$
$T \rightarrow \bullet F,$
$F \rightarrow \bullet (E),$
$F \rightarrow \bullet \textbf{id},$
$\}$

$I_1 = \text{Goto}(I_0, E) = \{$
$E' \rightarrow E\bullet,$
$E \rightarrow E \bullet + T$
$\}$

$I_2 = \text{Goto}(I_0, T) = \{$
$E \rightarrow T\bullet,$
$T \rightarrow T \bullet * F$
$\}$

$I_3 = \text{Goto}(I_0, F) = \{$
$T \rightarrow F\bullet$
$\}$

$I_5 = \text{Goto}(I_0, \textbf{id}) = \{$
$F \rightarrow \textbf{id}\bullet$
$\}$

$I_4 = \text{Goto}(I_0, \text{"("}) = \{$
$F \rightarrow (\bullet E),$
$E \rightarrow \bullet E + T,$
$E \rightarrow \bullet T,$
$T \rightarrow \bullet T * F,$
$T \rightarrow \bullet F,$
$F \rightarrow \bullet (E),$
$F \rightarrow \bullet \textbf{id},$
$\}$

$I_7 = \text{Goto}(I_2, *) = \{$
$T \rightarrow T * \bullet F,$
$F \rightarrow \bullet (E),$
$F \rightarrow \bullet \textbf{id}$
$\}$

Canonical Collection of Sets of LR(0) Items

$I_6 = \text{Goto}(I_1, +) = \{$
$E \rightarrow E + \bullet T,$
$T \rightarrow \bullet T * F,$
$T \rightarrow \bullet F,$
$F \rightarrow \bullet(E),$
$F \rightarrow \bullet\mathbf{id},$
$\}$

$I_8 = \text{Goto}(I_4, E) = \{$
$E \rightarrow E \bullet + T,$
$F \rightarrow (E \bullet)$
$\}$

$I_9 = \text{Goto}(I_6, T) = \{$
$E \rightarrow E + T \bullet,$
$T \rightarrow T \bullet * F$
$\}$

$I_{10} = \text{Goto}(I_7, F) = \{$
$T \rightarrow T * F \bullet,$
$\}$

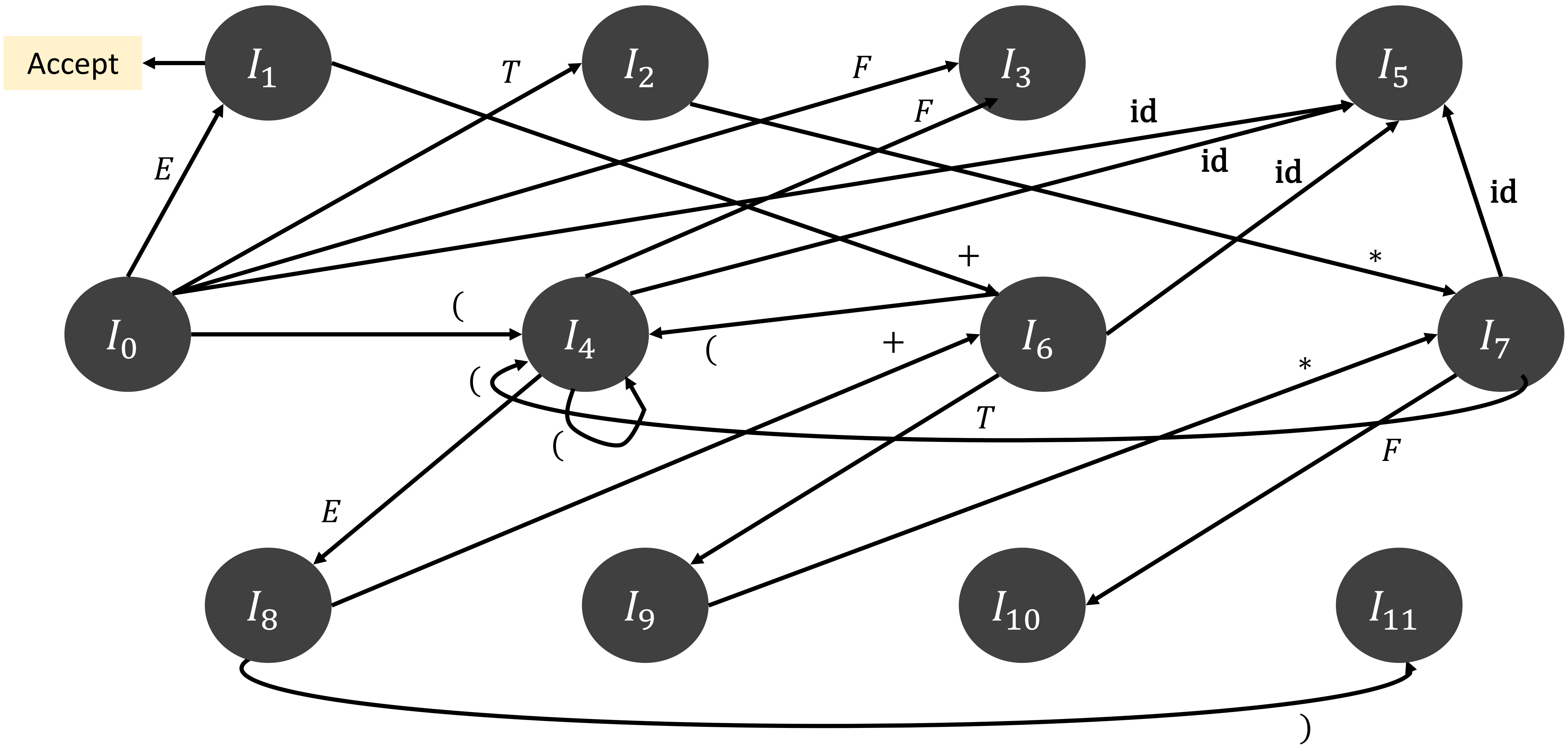
$I_{11} = \text{Goto}(I_8, ")") = \{$
$F \rightarrow (E) \bullet$
$\}$

$I_2 = \text{Goto}(I_4, T)$
$I_3 = \text{Goto}(I_4, F)$
$I_4 = \text{Goto}(I_4, "(")$
$I_5 = \text{Goto}(I_4, \mathbf{id})$
$I_3 = \text{Goto}(I_6, F)$
$I_4 = \text{Goto}(I_6, "(")$
$I_5 = \text{Goto}(I_6, \mathbf{id})$
$I_4 = \text{Goto}(I_7, "(")$
$I_5 = \text{Goto}(I_7, \mathbf{id})$
$I_6 = \text{Goto}(I_8, +)$
$I_7 = \text{Goto}(I_9, *)$

LR(0) Automaton

- An LR parser makes shift-reduce decisions by maintaining states
- Canonical LR(0) collection is used for constructing a DFA for parsing
- States represent sets of LR(0) items in the canonical LR(0) collection
  - Start state is Closure($\{S' \rightarrow \bullet S\}$), where S' is the start symbol of the augmented grammar
  - State j refers to the state corresponding to the set of items I_j

LR(0) Automaton



Use of LR(0) Automaton

- How can LR(0) automata help with shift-reduce decisions?
- Suppose string γ of grammar symbols takes the automaton from start state S_0 to state S_j
  - Shift on next input symbol a if S_j has a transition on a
  - Otherwise, reduce
    - Items in state S_j help decide which production to use

Shift-Reduce Parser with LR(0) Automaton

Stack	Symbols	Input	Action
0	$\$$	$\mathbf{id} * \mathbf{id}\$$	Shift to 5
$0\ 5$	$\$\mathbf{id}$	$* \mathbf{id}\$$	Reduce by $F \rightarrow \mathbf{id}$
$0\ 3$	$\$F$	$* \mathbf{id}\$$	Reduce by $T \rightarrow F$
$0\ 2$	$\$T$	$* \mathbf{id}\$$	Shift to 7
$0\ 2\ 7$	$\$T *$	$\mathbf{id}\$$	Shift to 5
$0\ 2\ 7\ 5$	$\$T * \mathbf{id}$	$\$$	Reduce by $F \rightarrow \mathbf{id}$
$0\ 2\ 7\ 10$	$\$T * F$	$\$$	Reduce by $T \rightarrow T * F$
$0\ 2$	$\$T$	$\$$	Reduce by $E \rightarrow T$
$0\ 1$	$\$E$	$\$$	Accept

Viable Prefix

- A viable prefix is a prefix of a right sentential form that can appear on the stack of a shift-reduce parser
  - α is a viable prefix if $\exists w$ such that αw is a right sentential form

$$E \rightarrow T \rightarrow T * F \rightarrow T * \mathbf{id} \rightarrow F * \mathbf{id} \rightarrow \mathbf{id} * \mathbf{id}$$

- $\mathbf{id} *$ is a prefix of a right sentential form, but it can never appear on the stack
  - Always reduce by $F \rightarrow \mathbf{id}$ before shifting $*$
  - Not all prefixes of a right sentential form can appear on the stack
- There is no error as long as the parser has viable prefixes on the stack

Example of a Viable Prefix

$S \rightarrow X_1X_2X_3X_4$
$A \rightarrow X_1X_2$

Let $w = X_1X_2X_3$

Stack	Input
$\$$	$X_1X_2X_3\$$
$\$X_1$	$X_2X_3\$$
$\$X_1X_2$	$X_3\$$
$\$A$	$X_3\$$
$\$AX_3$	$\$$

$X_1X_2X_3$ can never appear on a stack

Challenges with LR(0) Parsing

- An LR(0) parser works only if each state with a reduce action has only one possible reduce action and no shift action

$\{ \; L \rightarrow L, S\bullet \; \}$	$\{ \; L \rightarrow L, S\bullet$ $S \rightarrow S\bullet, L \; \}$	$\{ \; L \rightarrow S, L\bullet$ $L \rightarrow S\bullet \; \}$
Ok	Shift-reduce conflict	Reduce-reduce conflict

- Takes shift/reduce decisions without any lookahead token
  - Lacks the power to parse programming language grammars

Challenges with LR(0) Parsing

- Consider the following grammar for adding numbers

$$S \rightarrow S + E \mid E$$
$$E \rightarrow \textbf{num}$$

Left associative

$$S \rightarrow E + S \mid E$$
$$E \rightarrow \textbf{num}$$

Right associative

Challenges with LR(0) Parsing

- Consider the following grammar for adding numbers

$S \rightarrow S + E \mid E$ $E \rightarrow \mathbf{num}$
Left associative

$S \rightarrow E + S \mid E$ $E \rightarrow \mathbf{num}$
Right associative

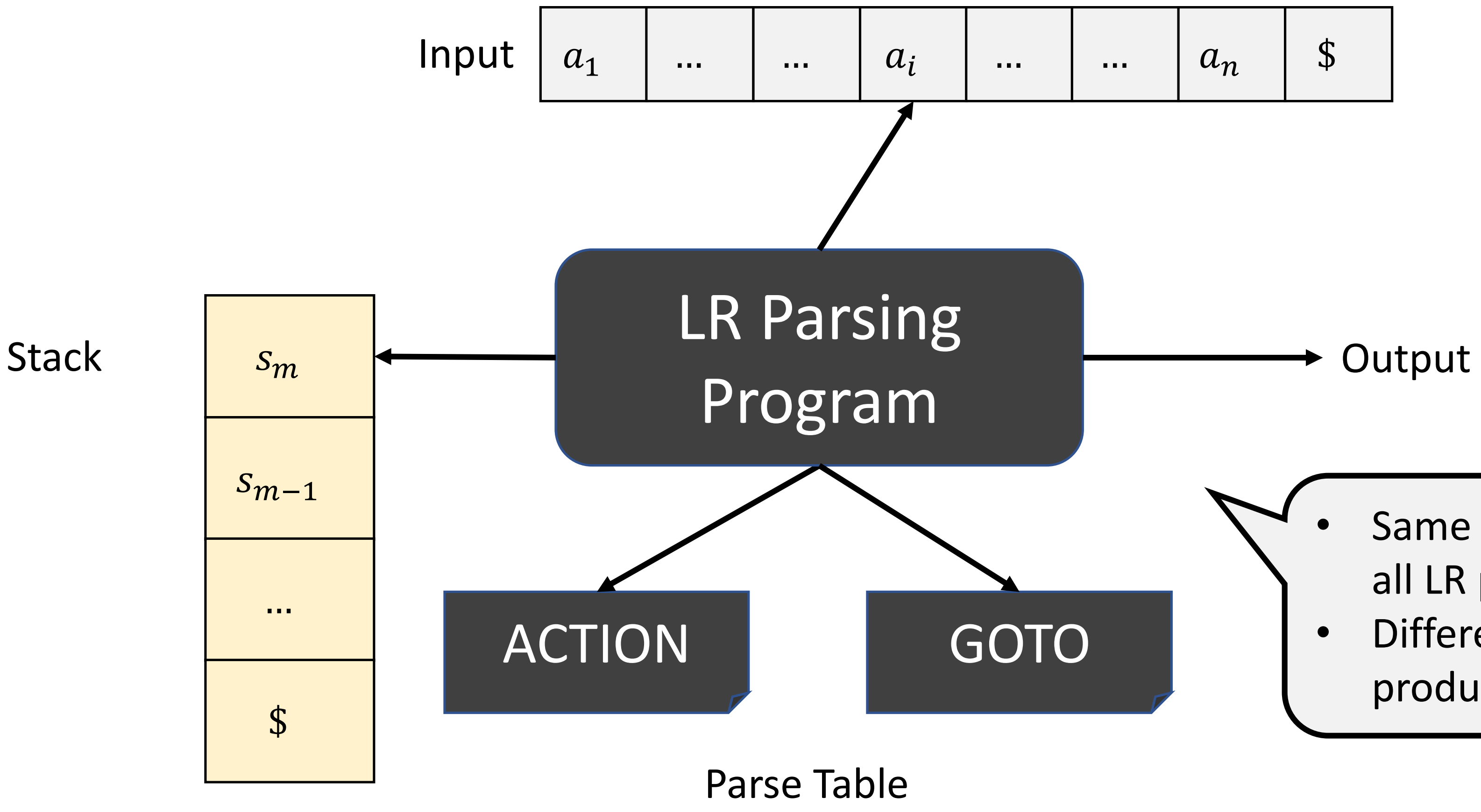
Not LR(0)

$S \rightarrow E \bullet + S$ $S \rightarrow E \bullet$
Shift-reduce conflict

Simple LR Parsing

SLR(1)

Block Diagram of LR Parser



- Same driver program is used for all LR parsers
- Different LR parsing techniques produce different parse tables

LR Parsing Algorithm

- The parser driver is same for all LR parsers
  - Only the parsing table changes across parsers
- A shift-reduce parser shifts a symbol, and an LR parser shifts a state
- By construction, all transitions to state j is for the same symbol X
  - Each state, except the start state, has a unique grammar symbol associated with it

SLR(1) Parsing

- Extends LR(0) parser to eliminate a few conflicts
  - Uses LR(0) items and LR(0) automaton
- For each reduction $A \to \beta$, look at the next symbol c
- Apply reduction only if $c \in \text{FOLLOW}(A)$ or $c = \epsilon$ and $S \overset{*}{\Rightarrow} \gamma A$

Structure of SLR Parsing Table

- Assume S_i is top of the stack and a_i is the current input symbol
- Parsing table consists of two parts: an Action and a Goto function
- Action table is indexed by state and terminal symbols
  - Action[S_i, a_i] can have four values
    - Shift a_i to the stack, goto state S_j
    - Reduce by rule k
    - Accept
    - Error (empty cell in the table)
- Goto table is indexed by state and nonterminal symbols

Constructing SLR Parsing Table

1) Construct LR(0) canonical collection $C = \{I_0, I_1, \ldots, I_n\}$ for grammar G'
2) State i is constructed from I_i
   a) If $[A \to \alpha\bullet a\beta]$ is in I_i and Goto(I_i, a) = I_j, then set Action[i, a] = "Shift j"
   b) If $[A \to \alpha\bullet]$ is in I_i, then set Action [i, a] = "Reduce $A \to \alpha$" for all a in $\mathrm{FOLLOW}(A)$
   c) If $[S' \to S\bullet]$ is in I_i, then set Action [i, \$] = "Accept"
3) If Goto(I_i, A) = I_j, then Goto[i, A] = j
4) All entries left undefined are "errors"

SLR Parsing for Expression Grammar

Rule #	Rule
1	$E \rightarrow E + T$
2	$E \rightarrow T$
3	$T \rightarrow T * F$
4	$T \rightarrow F$
5	$F \rightarrow (E)$
6	$F \rightarrow$ id

- sj means shift and stack state i
- rj means reduce by rule $\#j$
- acc means accept
- blank means error

SLR Parsing Table

State	Action						Goto		
	id	+	*	(	)	\$	E	T	F
0	$s5$			$s4$			1	2	3
1		$s6$				acc			
2		$r2$	$s7$		$r2$	$r2$			
3		$r4$	$r4$		$r4$	$r4$			
4	$s5$			$s4$			8	2	3
5		$r6$	$r6$		$r6$	$r6$			
6	$s5$			$s4$				9	3
7	$s5$			$s4$					10
8		$s6$			$s11$				
9		$r1$	$s7$		$r1$	$r1$			
10		$r3$	$r3$		$r3$	$r3$			
11		$r5$	$r5$		$r5$	$r5$			

LR Parser Configurations

- A LR parser configuration is a pair $<s_0, s_1, \ldots, s_m, \quad a_i a_{i+1} \ldots a_n \$>$
  - Left half is stack content, and right half is the remaining input
- Configuration represents the right sentential form $X_1 X_2 \ldots X_m a_i a_{i+1} \ldots a_n$

LR Parsing Algorithm

- If Action$[s_m, a_i]$ = shift s, new configuration is $<s_0, s_1, \ldots, s_m s, a_{i+1} \ldots a_n \$>$
- If Action$[s_m, a_i]$ = reduce $A \rightarrow \beta$, new configuration is $<s_0, s_1, \ldots, s_{m-r}, a_i a_{i+1} \ldots a_n \$>$
  - Assume r is $|\beta|$ and s = Goto$[s_{m-r}, A]$
- If Action$[s_m, a_i]$ = accept, parsing is successful
- If Action$[s_m, a_i]$ = error, parsing has discovered an error

LR Parsing Program

```
Let a be the first symbol of input w$
while (1)
    let s be the top of the stack
    if Action[a] == shift t
        push t onto the stack
        let a be the next input symbol
    else if Action[s, a] == reduce A → β
        pop |β| symbols off the stack
        push Goto[t, A] onto the stack
        output production A → β
    else if Action[s, a] == accept
        break
    else
        invoke error recovery
```

Moves of an LR Parser on $\mathbf{id} * \mathbf{id} + \mathbf{id}$

	Stack	Symbols	Input	Action
1	0		$\mathbf{id} * \mathbf{id} + \mathbf{id}\$$	Shift
2	0 5	$\mathbf{id}$	$* \mathbf{id} + \mathbf{id}\$$	Reduce by $F \rightarrow \mathbf{id}$
3	0 3	F	$* \mathbf{id} + \mathbf{id}\$$	Reduce by $T \rightarrow F$
4	0 2	T	$* \mathbf{id} + \mathbf{id}\$$	Shift
5	0 2 7	$T *$	$\mathbf{id} + \mathbf{id}\$$	Shift
6	0 2 7 5	$T * \mathbf{id}$	$+\mathbf{id}\$$	Reduce by $F \rightarrow \mathbf{id}$
7	0 2 7 10	$T * F$	$+\mathbf{id}\$$	Reduce by $T \rightarrow T * F$
8	0 2	T	$+\mathbf{id}\$$	Reduce by $E \rightarrow T$
9	0 1	E	$+\mathbf{id}\$$	Shift
10	0 1 6	$E +$	$\mathbf{id}\$$	Shift

Moves of an LR Parser on $\mathbf{id} * \mathbf{id} + \mathbf{id}$

	Stack	Symbols	Input	Action
11	$0\ 1\ 6\ 5$	$E + \mathbf{id}$	$\$$	Reduce by $F \rightarrow \mathbf{id}$
12	$0\ 1\ 6\ 3$	$E + F$	$\$$	Reduce by $T \rightarrow F$
13	$0\ 1\ 6\ 9$	$E + T$	$\$$	Reduce by $E \rightarrow E + T$
14	$0\ 1$	E	$\$$	Accept

Limitations of SLR Parsing

- If an SLR parse table for a grammar does not have multiple entries in any cell then the grammar is unambiguous
- Every SLR(1) grammar is unambiguous, but there are unambiguous grammars that are not SLR(1)

Limitations of SLR Parsing

Unambiguous grammar
$S \to L = R \mid R$ $L \to *R \mid \mathbf{id}$ $R \to L$

Example Derivation

$S \Rightarrow L = R \Rightarrow *R = R$

$\text{FIRST}(S) = \text{FIRST}(L) = \text{FIRST}(R) = \{*, \mathbf{id}\}$

$\text{FOLLOW}(S) = \text{FOLLOW}(L) = \text{FOLLOW}(R) = \{=, \$\}$

Canonical LR(0) Collection

I_0 = Closure($S' \rightarrow . S$)= {
$S' \rightarrow \bullet S,$
$S \rightarrow \bullet L = R,$
$S \rightarrow \bullet R,$
$L \rightarrow \bullet * R,$
$L \rightarrow \bullet \mathbf{id},$
$R \rightarrow \bullet L$
}
I_1 = Goto(I_0, S) = {
$S' \rightarrow S\bullet$
}
I_2 = Goto(I_0, L)= {
$S \rightarrow L\bullet = R,$
$R \rightarrow L\bullet$
}

I_3 = Goto(I_0, R)= {
$S \rightarrow R\bullet$
}
I_4 =Goto(I_0, R)= {
$L \rightarrow * \bullet R,$
$R \rightarrow \bullet L,$
$L \rightarrow \bullet * R,$
$L \rightarrow \bullet \mathbf{id}$
}
I_6 = Goto(I_2, '=')= {
$S \rightarrow L = \bullet R,$
$R \rightarrow \bullet L,$
$L \rightarrow \bullet * R,$
$L \rightarrow \bullet \mathbf{id}$
}

I_5 = Goto($I_0, \mathbf{id}$)= {
$L \rightarrow \bullet \mathbf{id}$
}
I_7 =Goto(I_4, R)= {
$L \rightarrow * R\bullet$
}
I_8 = Goto(I_4, L)= {
$R \rightarrow L\bullet$
}
I_9 = Goto(I_6, R)= {
$S \rightarrow L = R\bullet$
}

SLR Parsing Table

State	Action				Goto		
	=	$*$	id	\$	S	L	R
0		$s4$	$s5$		1	2	3
1				acc			
2	$s6, r6$			$r6$			
3							
4		$s4$	$s5$			8	7
5	$r5$			$r5$			
6		$s4$	$s5$			8	9
7	$r4$			$r4$			
8	$r6$			$r6$			
9				$r2$			

Shift-Reduce Conflict with SLR Parsing

I_0 = Closure($S' \rightarrow . S$)= {
$S' \rightarrow \bullet S,$
$S \rightarrow \bullet L = R,$
$S \rightarrow \bullet R$

}
I_1 = G

$S' \rightarrow S\bullet$
}
I_2 = Goto(I_0, L)= {
$S \rightarrow L\bullet = R,$
$R \rightarrow L\bullet$
}

I_3 = Goto(I_0, R)= {
$S \rightarrow R\bullet$
}

I_6 = Goto(I_2, '=')= {
$S \rightarrow L = \bullet R,$
$R \rightarrow \bullet L,$
$L \rightarrow \bullet * R,$
$L \rightarrow \bullet\mathbf{id}$
}

I_5 = Goto(I_0, **id**)= {
$L \rightarrow \bullet\mathbf{id}$
}

}
I_9 = Goto(I_6, R)= {
$S \rightarrow L = R\bullet$
}

1. Action[2,=] = Shift 6
2. Action[2,=] = Reduce $R \rightarrow L$ since $'='\ \in \mathrm{FOLLOW}(R)$

Moves of an LR Parser on **id=id**

Stack	Input	Action
0	**id=id**\$	Shift 5
0 **id** 5	=**id**\$	Reduce by $L \rightarrow$ **id**
0 L 2	=**id**\$	Reduce by $R \rightarrow L$
0 R 3	=**id**\$	Error

Stack	Input	Action
0	**id=id**\$	Shift 5
0 **id** 5	=**id**\$	Reduce by $L \rightarrow$ **id**
0 L 2	=**id**\$	Shift 6
0 L 2 = 6	**id**\$	Shift 5
0 L 2 = 6 **id** 5	\$	Reduce by $L \rightarrow$ **id**
0 L 2 = 6 L 8	\$	Reduce by $R \rightarrow L$
0 L 2 = 6 R 9	\$	Reduce by $S \rightarrow L = R$
0 S 1	\$	Accept

Moves of an LR Parser on **id=id**

- State i calls for a reduction by $A \rightarrow \alpha$ if the set of items I_i contains item $[A \rightarrow \alpha\bullet]$ and $a \in \mathrm{FOLLOW}(A)$

- Suppose βA is a viable prefix on top of the stack
- There may be no right sentential form where a follows βA
  - No right sentential form begins with $R = \cdots$
  - ➢ Parser should not reduce by $A \rightarrow \alpha$

$0\ L\ 2 = 6\ R\ 9$	\$	Reduce by $S \rightarrow L = R$
$0\ S\ 1$	\$	Accept

Moves of an LR Parser on **id=id**

Stack	Input	Action
0	**id**=**id**\$	Shift 5

Stack	Input	Action
0	**id**=**id**\$	Shift 5
$0\ L\ 2 = 6\ \mathbf{id}\ 5$	\$	Reduce by $L \rightarrow \mathbf{id}$
$0\ L\ 2 = 6\ L\ 8$	\$	Reduce by $R \rightarrow L$
$0\ L\ 2 = 6\ R\ 9$	\$	Reduce by $S \rightarrow L = R$
$0\ S\ 1$	\$	Accept

SLR parsers cannot remember the left context

- SLR(1) states only tell us about the sequence on top of the stack, not what is below on the stack

Canonical LR Parsing

LR(1) Item

- An LR(1) item of a CFG G is a string of the form $[A \rightarrow \alpha{\bullet}\beta, a]$
  - $A \rightarrow \alpha\beta$ is a production in G, and $a \in T \cup \{\$\}$
  - There is now one symbol lookahead
- Suppose $[A \rightarrow \alpha{\bullet}\beta, a]$ where $\beta \neq \epsilon$, then the lookahead does not help
- If $[A \rightarrow \alpha\bullet, a]$, reduce only if next input symbol is a
  - Set of possible terminals will always be a subset of $\mathrm{FOLLOW}(A)$, but can be a proper subset

LR(1) Item

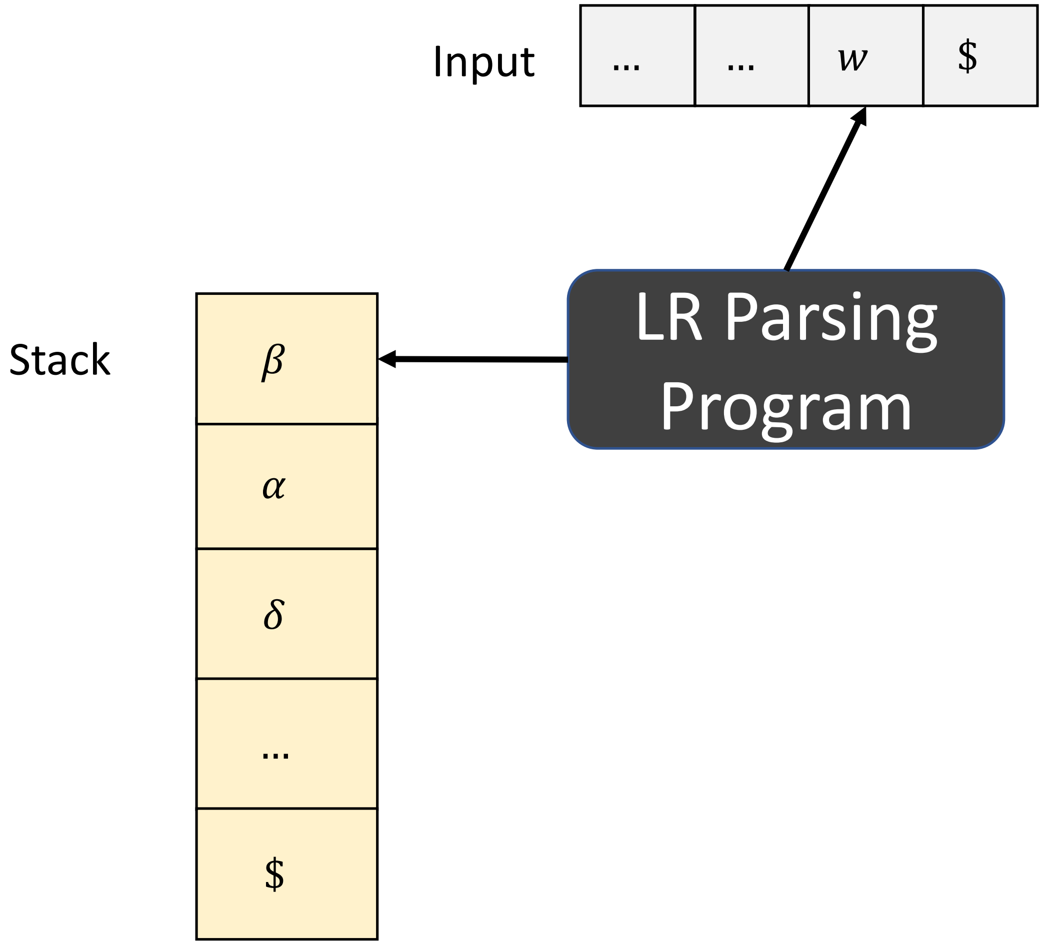
- An LR(1) item $[A \rightarrow \alpha \bullet \beta, a]$ is valid for a viable prefix γ if there is a derivation

$$S \underset{rm}{\Rightarrow}^{*} \delta A w \underset{rm}{\Rightarrow} \delta \alpha \beta w$$

where

i. $\gamma = \delta a$, and

ii. a is first symbol of w or $w = \epsilon$ and $a = \$$



Constructing LR(1) Sets of Items

Closure(I)	**Goto(I, X)**

repeat
    for each item $[A \to \alpha \bullet B\beta, a]$ in I
        for each production $B \to \gamma$ in G'
            for each terminal b in $\mathrm{FIRST}(\beta a)$
                add $[B \to \bullet\gamma, b]$ to set I
until no more items are added to I
return I
	initialize J to be the empty set
	for each item $[A \to \alpha \bullet X\beta, a]$ in I
	    add item $[A \to \alpha X \bullet \beta, a]$ to set J
	return Closure(J)

Constructing LR(1) Sets of Items

Items(G')

    C = Closure({$[S' \rightarrow \bullet S, \$]$})

    repeat

    for each set of items I in C

        for each grammar symbol X

            if Goto(I, X) $\neq \phi$ and Goto(I, X) $\notin C$

                add Goto(I, X) to C

    until no new sets of items are added to C

Example Construction of LR(1) Items

Rule #	Production
0	$S' \rightarrow S$
1	$S \rightarrow CC$
2	$C \rightarrow cC$
3	$C \rightarrow d$

generates the regular language c^*dc^*d

I_0 = Closure($[S' \rightarrow \bullet S, \$]$)= {
$S' \rightarrow \bullet S, \$,$
$S \rightarrow \bullet CC, \$,$
$C \rightarrow \bullet cC, c/d,$
$C \rightarrow \bullet d, c/d$
}

I_1 = Goto(I_0, S) = {
$S' \rightarrow S\bullet, \$$
}

Example Construction of LR(1) Items

$I_0 =$ Closure($[S' \rightarrow .S, \$]$)= {
$S' \rightarrow \bullet S, \$,$
$S \rightarrow \bullet CC, \$,$
$C \rightarrow \bullet cC, c/d,$
$C \rightarrow \bullet d, c/d$
}

$I_1 =$ Goto(I_0, S) = {
$S' \rightarrow S\bullet, \$$
}

$I_2 =$ Goto(I_0, C)= {
$S \rightarrow C\bullet C, \$,$
$C \rightarrow \bullet cC, \$,$
$C \rightarrow \bullet d, \$$
}

$I_3 =$ Goto(I_0, c)= {
$C \rightarrow c\bullet C, c/d,$
$C \rightarrow \bullet cC, c/d,$
$C \rightarrow \bullet d, c/d$
}

$I_4 =$ Goto(I_0, d) = {
$C \rightarrow d\bullet, c/d$
}

$I_5 =$ Goto(I_2, C) = {
$C \rightarrow CC\bullet, \$$
}

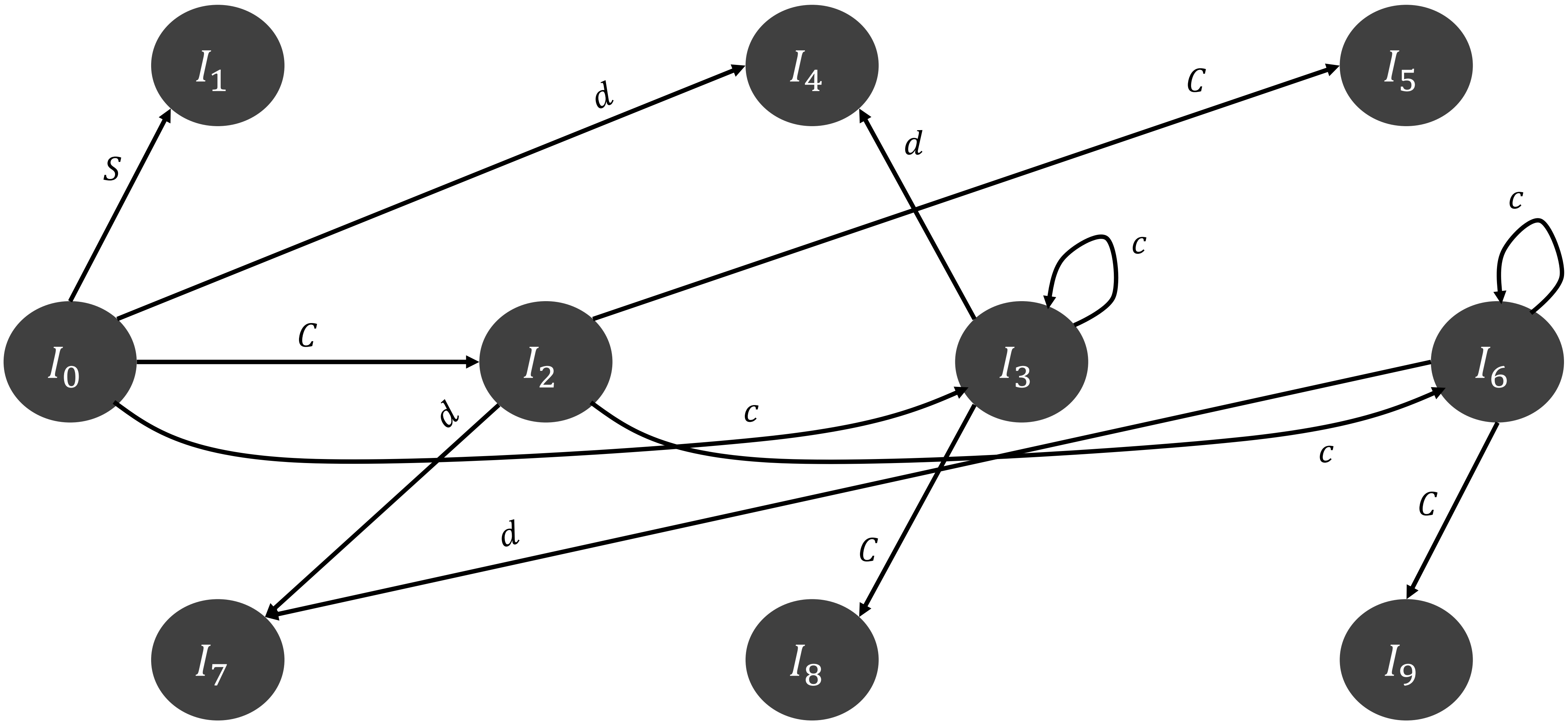
$I_6 =$ Goto(I_2, c)= {
$C \rightarrow c\bullet C, \$,$
$C \rightarrow \bullet cC, \$,$
$C \rightarrow \bullet d, \$$
}

$I_7 =$ Goto(I_2, d) = {
$C \rightarrow d\bullet, \$$
}

$I_8 =$ Goto(I_3, C) = {
$C \rightarrow cC\bullet, c/d$
}

$I_9 =$ Goto(I_6, C)= {
$C \rightarrow cC\bullet, \$$
}

LR(1) Automaton



Construction of Canonical LR(1) Parsing Tables

- Construct $C' = \{I_0, I_1, \dots, I_n\}$
- State i of the parser is constructed from I_i
  - If $[A \rightarrow \alpha \bullet a\beta, b]$ is in I_i and Goto(I_i,a) = I_j, then set Action[i, a]="shift j"
  - If $[A \rightarrow \alpha \bullet, a]$ is in I_i, $A \neq S'$, then set Action[i,a]="reduce $A \rightarrow \alpha \bullet$"
  - If $[S' \rightarrow S \bullet, \$]$ is in I_i, then set Action[i,\$]="accept"
- If Goto(I_i, A)= I_j, then Goto[i, A] = j
- Initial state of the parser is constructed from the set of items containing $[S' \rightarrow \bullet S, \$]$

Canonical LR(1) Parsing Table

State	Action			Goto	
	c	d	$\$$	S	C
0	$s3$	$s4$		1	2
1			acc		
2	$s6$	$s7$			5
3	$s3$	$s4$			8
4	$r3$	$r3$			
5			$r1$		
6	$s6$	$s7$			9
7			$r3$		
8	$r2$	$r2$			
9			$r2$		

Canonical LR(1) Parsing

- If the parsing table has no multiply-defined cells, then the corresponding grammar G is LR(1)

- Every SLR(1) grammar is an LR(1) grammar
  - Canonical LR parser may have more states than SLR

LALR Parsing

Example Construction of LR(1) Items

$I_0 =$ Closure($[S' \rightarrow .S, \$]$)= {
$S' \rightarrow \bullet S, \$,$
$S \rightarrow \bullet CC, \$,$
$C \rightarrow \bullet cC, c/d,$
$C \rightarrow \bullet d, c/d$
}

$I_1 =$ Goto(I_0, S) = {
$S' \rightarrow S\bullet, \$$
}

$I_2 =$ Goto(I_0, C)= {
$S \rightarrow C\bullet C, \$,$
$C \rightarrow \bullet cC, \$,$
$C \rightarrow \bullet d, \$$
}

$I_3 =$ Goto(I_0, c)= {
$C \rightarrow c\bullet C, c/d,$
$C \rightarrow \bullet cC, c/d,$
$C \rightarrow \bullet d, c/d$
}

$I_4 =$ Goto(I_0, d) = {
$C \rightarrow d\bullet, c/d$
}

$I_5 =$ Goto(I_2, C) = {
$C \rightarrow CC\bullet, \$$
}

$I_6 =$ Goto(I_2, c)= {
$C \rightarrow c\bullet C, \$,$
$C \rightarrow \bullet cC, \$,$
$C \rightarrow \bullet d, \$$
}

$I_7 =$ Goto(I_2, d) = {
$C \rightarrow d\bullet, \$$
}

$I_8 =$ Goto(I_3, C) = {
$C \rightarrow cC\bullet, c/d$
}

$I_9 =$ Goto(I_6, C)= {
$C \rightarrow cC\bullet, \$$
}

I_3 and I_6, I_4 and I_7, and I_8 and I_9 only differ in the second components

Lookahead LR (LALR) Parsing

- CLR(1) parser has a large number of states
- Lookahead LR (LALR) parser
  - Merge sets of LR(1) items that have the same core, i.e., first component
    - A core is a set of LR(0) items
  - LALR parser is used in many parser generators (for e.g., Yacc and Bison)
    - Fewer number of states, same as SLR

Construction of LALR Parsing Table

- Construct $C = \{I_0, I_1, \dots, I_n\}$, the collection of sets of LR(1) items
- For each core present in LR(1) items, find all sets having the same core and replace these sets by their union
- Let $C' = \{J_0, J_1, \dots, J_n\}$ be the resulting sets of LR(1) items
  - Also called LALR collection
- Construct Action table as was done earlier, parsing actions for state i is constructed from J_i
- Let $J = I_1 \cup I_2 \cup \cdots \cup I_k$, where the cores of $I_1, I_2, \dots, I_k$ are same.
  - Cores of Goto(I_1, X), Goto(I_2, X), …, Goto(I_k, X) will also be the same.
  - Let $K =$ Goto(I_1, X) $\cup$ Goto(I_2, X) $\cup$ … $\cup$ Goto(I_k, X), then Goto(J,X) = K

LALR Grammar

- If there are no parsing action conflicts, then the grammar is LALR(1)

Rule #	Production
0	$S' \rightarrow S$
1	$S \rightarrow CC$
2	$C \rightarrow cC$
3	$C \rightarrow d$

I_{36} = Goto(I_0, c)= {
$C \rightarrow c{\bullet}C, c/d/\$,$
$C \rightarrow {\bullet}cC, c/d/\$,$
$C \rightarrow {\bullet}d, c/d/\$$
}

I_{47} = Goto(I_0, d) = {
$C \rightarrow d{\bullet}, c/d/\$$
}

I_{89} = Goto(I_3, C) = {
$C \rightarrow cC{\bullet}, c/d/\$$
}

LALR Parsing Table

State	Action			Goto	
	c	d	$\$$	S	C
0	$s36$	$s47$		1	2
1			acc		
2	$s36$	$s47$			5
36	$s36$	$s47$			89
47	$r3$	$r3$	$r3$		
5			$r1$		
89	$r2$	$r2$	$r2$		

Notes on LALR Parsing Table

- Modified parser behaves as original
- Merging items can **never** produce shift/reduce conflicts
  - Suppose there is a conflict on lookahead a
  - Shift due to item $[B \rightarrow \beta \bullet a\gamma, b]$ and reduce due to item $[A \rightarrow \alpha \bullet, a]$
  - But merged state was formed from states with same cores
- Merging items **may** produce reduce/reduce conflicts

Reduce-Reduce Conflicts due to Merging

LR(1) grammar
$S' \to S$ $S \to aAd \mid bBd \mid aBe \mid bAe$ $A \to c$ $B \to c$
acd, ace, bcd, bce

$\{[A \to c\bullet, d],\ [B \to c\bullet, e]\}$ $\qquad$ $\{[A \to c\bullet, e],\ [B \to c\bullet, d]\}$

$\{[A \to c\bullet, d/c\ ],\ [B \to c\bullet, d/e]\}$

Dealing with Errors with LALR Parsing

#	Production
0	$S' \rightarrow S$
1	$S \rightarrow CC$
2	$C \rightarrow cC$
3	$C \rightarrow d$

- Consider an erroneous input ccd

CLR Parsing Table

State	Action			Goto	
	c	d	$\$$	S	C
0	$s3$	$s4$		1	2
1			acc		
2	$s6$	$s7$			5
3	$s3$	$s4$			8
4	$r3$	$r3$			
5			$r1$		
6	$s6$	$s7$			9
7			$r3$		
8	$r2$	$r2$			
9			$r2$		

LALR Parsing Table

State	Action			Goto	
	c	d	$\$$	S	C
0	$s36$	$s47$		1	2
1			acc		
2	$s36$	$s47$			5
36	$s36$	$s47$			89
47	$r3$	$r3$	$r3$		
5			$r1$		
89	$r2$	$r2$	$r2$		

Dealing with Errors with LALR Parsing

- Consider an erroneous input ccd

CLR Parsing Table

State	Action			Goto	
	c	d	$\$$	S	C
7			$r3$		
8	$r2$	$r2$			
9			$r2$		

LALR Parsing Table

State	Action			Goto	
	c	d	$\$$	S	C
89	$r2$	$r2$	$r2$		

- CLR parser will not even reduce before reporting an error
- SLR and LALR parsers may reduce several times before reporting an error
  - Will never shift an erroneous input symbol onto the stack

Using Ambiguous Grammars

Dealing with Ambiguous Grammars

$$E' \rightarrow E$$
$$E \rightarrow E + E \mid E * E \mid (E) \mid \mathbf{id}$$

$I_0 = \text{Closure}(\{E' \rightarrow \bullet E\}) = \{$
$E' \rightarrow \bullet E,$
$E \rightarrow \bullet E + E,$
$E \rightarrow \bullet E * E,$
$E \rightarrow \bullet (E),$
$E \rightarrow \bullet \mathbf{id}$
$\}$

$I_1 = \text{Goto}(I_0, E) = \{$
$E' \rightarrow E \bullet,$
$E \rightarrow E \bullet + E,$
$E \rightarrow E \bullet * E$
$\}$

$I_2 = \text{Goto}(I_0, '(') = \{$
$E \rightarrow (\bullet E),$
$E \rightarrow \bullet E + E,$
$E \rightarrow \bullet E * E,$
$E \rightarrow \bullet (E),$
$E \rightarrow \bullet \mathbf{id}$
$\}$

$I_3 = \text{Goto}(I_0, \mathbf{id}) = \{$
$E \rightarrow \mathbf{id} \bullet$
$\}$

$I_4 = \text{Goto}(I_0, '+') = \{$
$E \rightarrow E + \bullet E,$
$E \rightarrow \bullet E + E,$
$E \rightarrow \bullet E * E,$
$E \rightarrow \bullet (E),$
$E \rightarrow \bullet \mathbf{id}$
$\}$

$I_9 = \text{Goto}(I_6, ')') = \{$
$E \rightarrow (E) \bullet$
$\}$

$I_5 = \text{Goto}(I_0, '*') = \{$
$E \rightarrow E * \bullet E,$
$E \rightarrow \bullet E + E,$
$E \rightarrow \bullet E * E,$
$E \rightarrow \bullet (E),$
$E \rightarrow \bullet \mathbf{id}$
$\}$

$I_6 = \text{Goto}(I_2, E) = \{$
$E \rightarrow (E \bullet),$
$E \rightarrow E \bullet + E,$
$E \rightarrow E \bullet * E,$
$\}$

$I_7 = \text{Goto}(I_4, E) = \{$
$E \rightarrow E + E \bullet,$
$E \rightarrow E \bullet + E,$
$E \rightarrow E \bullet * E,$
$\}$

$I_8 = \text{Goto}(I_5, E) = \{$
$E \rightarrow E * E \bullet,$
$E \rightarrow E \bullet + E,$
$E \rightarrow E \bullet * E$
$\}$

SLR(1) Parsing Table

State		Action					Goto
	id	$+$	$*$	$($	$)$	$\$$	E
0	$s3$			$s2$			1
1		$s4$	$s5$			acc	
2	$s3$			$s2$			6
3		$r4$	$r4$		$r4$	$r4$	
4	$s3$			$s2$			7
5	$s3$			$s2$			8
6		$s4$	$s5$		$s9$		
7		$s4, r1$	$s5, r1$		$r1$	$r1$	
8		$s4, r2$	$s5, r2$		$r2$	$r2$	
9		$r3$	$r3$		$r3$	$r3$	

Moves of an SLR Parser on $\mathbf{id} + \mathbf{id} * \mathbf{id}$

	Stack	Symbols	Input	Action
1	0		$\mathbf{id} + \mathbf{id} * \mathbf{id}\$$	Shift 3
2	0 3	$\mathbf{id}$	$+\mathbf{id} * \mathbf{id}\$$	Reduce by $E \rightarrow \mathbf{id}$
3	0 1	E	$+\mathbf{id} * \mathbf{id}\$$	Shift 4
4	0 1 4	$E +$	$\mathbf{id} * \mathbf{id}\$$	Shift 3
5	0 1 4 3	$E + \mathbf{id}$	$* \mathbf{id}\$$	Reduce by $E \rightarrow \mathbf{id}$
6	0 1 4 7	$E + E$	$* \mathbf{id}\$$	

SLR(1) Parsing Table

State	Action						Goto
	id	$+$	$*$	(	)	\$	E
0	$s3$			$s2$			1
1		$s4$	$s5$			acc	
2	$s3$			$s2$			6
3		$r4$	$r4$		$r4$	$r4$	
4	$s3$			$s2$			7
5	$s3$			$s2$			8
6		$s4$	$s5$		$s9$		
7		$s4, \mathbf{r1}$	$\mathbf{s5}, r1$		$r1$	$r1$	
8		$s4, \mathbf{r2}$	$s5, \mathbf{r2}$		$r2$	$r2$	
9		$r3$	$r3$		$r3$	$r3$	

Summary

Comparisons across Techniques

- SLR(1) = LR(0) items + FOLLOW
    - SLR(1) parsers can parse a larger number of grammars than LR(0)
    - Any grammar that can be parsed by an LR(0) parser can be parsed by an SLR(1) parser

- SLR(1) $\leq$ LALR(1) $\leq$ LR(1)
- SLR(k) $\leq$ LALR(k) $\leq$ LR(k)
- LL(k) $\leq$ LR(k)
- Ambiguous grammars are not LR

Summary

- Bottom-up parsing is a more powerful technique compared to top-down parsing
  - LR grammars can handle left recursion
  - Detects errors as soon as possible, and allows for better error recovery
- Automated parser generators such as Yacc and Bison

References

- A. Aho et al. Compilers: Principles, Techniques, and Tools, 2nd edition, Chapter 4.5-4.8.
- K. Cooper and L. Torczon. Engineering a Compiler, 2nd edition, Chapter 3.4.